(2) n^{th} derivative of $e^{ax}\cos(bx+c)$

Let $y = e^{ax}\cos(bx+c)$

$$y_1 = e^{ax}\left[-b\sin(bx+c)\right] + ae^{ax}\cos(bx+c)$$

Putting $\boxed{a = r\cos\theta}$ and $\boxed{b = r\sin\theta}$, we get

$$y_1 = re^{ax}\{\cos\theta\cos(bx+c) - \sin\theta\sin(bx+c)\}$$

$$= re^{ax}\cos(bx+c+\theta)$$

Similarly $y_2 = r^2 e^{ax}\cos(bx+c+2\theta)$

. . . .

$$\therefore y_n = r^n e^{ax}\cos(bx+c+n\theta)$$

$$\therefore y_n = (a^2+b^2)^{\frac{n}{2}} \cdot e^{ax} \cos\left[bx+c+n\tan^{-1} b/a\right]$$

i.e., $D^n\left[e^{ax}\cos(bx+c)\right] = (a^2+b^2)^{n/2} e^{ax}\cos\left(bx+c+n\tan^{-1} b/a\right)$

if $a=1, b=1, c=0$, then

$$y = e^x\cos x \text{ and } y_n = 2^{n/2} e^x \cos(x + n(\pi/4))$$

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(3) n^{th} derivative $y = \cos 2x \cos 3x = \cos 3x + \cos 2x$

$$= \frac{1}{2}\{\cos(3x+2x) + \cos(3x-2x)\}$$

[By using $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$]

$$= \frac{1}{2}\left[\cos 5x + \cos x\right]$$

$$y = \frac{1}{2}\cos 5x + \frac{1}{2}\cos x$$

Differentiating n times, we get.

$$y_n = \frac{5^n}{2}\cos\left[5x + \frac{n\pi}{2}\right] + \frac{1}{2}\cos\left[x + \frac{n\pi}{2}\right]$$